BOREL ON VAGUENESS

1. Overview

In 1907 Borel published a remarkable essay on the paradox of the heap entitled "Un paradoxe économique: le sophisme du tas de blé et les vérités statistiques", published in *la Revue du Mois*, a journal founded with his wife Camille Marbo in 1905. In that paper Borel proposes what is likely the first extended account of vagueness published in the twentieth century, and also the first statistical account of vagueness, with particular emphasis on the practical implications of the sorites paradox, including in economics. Borel's paper was later integrated in his book *Le Hasard*, published 1914, but has gone mostly unnoticed since its publication. Borel's paper did not have the impact it might have had otherwise, in particular on fuzzy logic. For instance, the theory of M. Black (1937), proposed thirty years later and also based on statistical considerations, had a real influence on the development of fuzzy logic, as testified by the writings of Goguen (1969) and Zadeh (1975), but Black too did not seem to be aware of Borel's work.

The aim of the present paper is to give a presentation of the historical context of Borel's paper, to highlight its persistent originality, and to draw some comparisons with more recent theories of vagueness and the heap paradox. Our paper is based on a parallel editorial project, including the completion of a translation of Borel's text in English, intended to make Borel's paper accessible to a large readership.

2. Granularity, decision-making and error theory

Borel's account of vagueness is original in at least three ways: first of all, Borel considers all soritical situations essentially as problems of individual decision-making. For Borel, in particular, lexical vagueness is only one manifestation of the problem of vagueness among many. For instance, Borel saw the most interesting manifestation of the heap paradox in the economic problem of the relation between variations of wholesale price and variations of retail price. All examples reviewed by Borel thus have the same abstract structure, namely: how exactly does a small variation of some input parameter along a given scale of measurement affect the inclusion of an object within a given category, when the scale relevant for the output category has a coarser granularity?

Secondly, Borel's account is driven both by the idea that there are no absolute boundaries for vague predicates (there is no least number of grains that makes a heap on his account), and by the rejection of the major premise of the sorites (which induces the paradox). For Borel, probabilities are the only plausible way of accommodating both the lack of sharp discontinuities for vague categories, but also of acknowledging the fact that subjects do set boundaries in concrete situations. For instance, Borel considers as a psychological illusion

the idea that if an individual is willing to pay some good n cents, he will remain equally willing to pay the same good $n + 1$ cents. Both individually and collectively, however, the propensity to switch categories corresponds to a probability coefficient on his account.

Thirdly, and more importantly, Borel establishes a central connection in his account between vagueness and imprecision. The core of Borel's discussion concerns the way in which rules of precise categorization get to be relaxed in practice. Borel gives the example of the rule for a publisher to review all and only manuscripts whose number of characters lies in a certain interval specified by precise numbers. The point of Borel's model is to show that the decision to process a manuscript will be a function of the error with which the number of characters of the manuscript is estimated. Borel in 1907 handles this example by an application of what he calls the *law of deviations* (loi des écarts), corresponding to the Gaussian law quantifiying the probability of deviating from the mean value of a random variable by more than a given magnitude (viz. Borel, Deltheil, Huron 1954). This law of deviation, which was refined later by P. Lévy and Lindeberg, is what we know today as the "central limit theorem" in statistics.

In the paper, we examine the details of Borel's account of imprecision as predicted by the theory of measurement errors. We also establish some connections with Borel's account of the sorites and two other papers written by Borel at the same period. The first is a paper by Borel on the notion of physical continuum, where Borel discusses the views of Poincaré on the definition of the physical continuum. Poincaré, aware of Fechner's work in psychophysics, was particularly concerned with the problem of intransitivities characteristic of psychophysical measurement (a weight A can be felt as heavy as B, B as heavy as C, but A be felt lighter than C). Borel (1909) outlines the view that such intransitivities can be resolved provided sufficiently many measurements can be taken. Some relevant details appear in another paper of his published in 1908 in the psychology journal *L'Année psychologique*. In that paper Borel discusses data presented by Titchener to illustrate Fechner's method of constant stimuli, and discusses the way in which the value of the differential threshold can be computed relative to a series of weights. These two papers, in combination with Borel's 1907 on the heap paradox, show that Borel saw a very fundamental relation between problems of categorization and foundational issues about precision in measurement.

3. Perspectives and legacy

In the last section of the paper, we conclude our presentation of Borel's account of vagueness with a comparison with some more recent degree-theoretic and statistical accounts of vagueness. Not surprisingly, the idea of dealing with vagueness by means of probabilities and statistical methods has been proposed a number of time since 1907, starting with Black's work, and including recently with attempts to establish a closer articulation between lexical semantics and probabilistic semantics (Lassiter 2011, Solt 2011), or to enrich degree-based logics (MacFarlane 2010, Smith 2008), supervaluationist accounts of vagueness (Simons 2010), or more generally to connect vagueness with either imperfect discrimination (Egré and Bonnay 2010, Fults 2010), ambiguity (Egré 2011) or stochastic

behavior (Franke et al. 2010). In the area of psychology, probabilistic models of categorization and discrimination have been more widespread ever since the birth of psychophysics (see in particular Hampton 2007 on threshold models for categorization), but mostly because probability theory has established itself as an essential tool for more than a century now. At the time Borel was writing on the heap, however, the idea that probability could be fruitfully applied to psychology was not as entrenched, even though it was acquiring momentum, in particular with the extraordinary development of psychophysics, and with the bridge drawn by scientists such as Quetelet, Galton and Pearson between physics and the social sciences (all three authors being referred to in Borel 1914).

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