Supervaluations, Subvaluations and indeterminism.

Genuine indeterminism states that the present moment is not able to completely determine what will be the case. This at least is the way in which Nuel Belnap and Mitchell Green outlined it (see Belnap and Green 1994, pp. 367-370). In other words, there is at least a fact E such that its happening or failing to happen in some (every) later moment is not determined by the present moment being so-and-so. Thus presented, genuine indeterminism is then a metaphysical position and must not be confused with an epistemic position: it does not state that future events are determined, though we do not know whether positively or negatively. This view is today among the most prominent forms of indeterminism, and we shall call it "indeterminism" simply.

From the point of view of model theory, tree-like structures (with branching on the right) are nowadays considered as being the most genuine semantic expression of this stance. But there is a large range of temporal logics that can be interpreted on them. Among these, supervaluationist approaches, like the one presented in (Thomason 1970) and (Thomason 1984) are nowadays the most widespread logical tools for reasoning in a genuine indeterministic framework.

Supervaluationism holds that, since the future is undetermined, and as a consequence of this, statements about the future, like for example "There will be a sea battle" or its negation, may be neither true nor false. Obviously, not all the sentences about the future should encounter this fate. "Either there will be a sea battle, or there will be no such battle" is true now, since no development of the events includes later moments where it fails to hold. More generally, all the valid sentences of classical logics - such as "A or not - A" - and their temporalisations hold at any moment of any development, and hence do not face the problem of "There will be a sea battle": they are true (if they are classically valid sentences) or false (if they are negations of such sentences). In a word, the laws of logic are settled, and the same plausibly applies to the laws of nature: In other words "Truth and falsehood imply settledness. Without settledness, there is lack of truth-value".

Here, though agreeing with the indeterministic stances that characterize supervaluationism, i.e that the future is open, we want to investigate a quite different view: the view that the future is - as we will say - abundant: statements about the future do not lack truth-value, instead they may well be glutty, that is both true and false. Truths and falsehoods about the future do not lack, but on the contrary abound. Not only "There will be a sea battle" is true today: also "There will be no sea battle" is, since the world is open to develop either way. We shall call such a position "the abundance of the future". It originates from a conception of the truth of statements about the future according to which any sentence A is true at a moment m if and only if there is at least a possible development of the events where A is true at a moment m', with m < m'.

This approach is related to supervaluations: it is indeed its dual. But, contrary to the former, abundance has never been seriously explored as a semantic for temporal logic, nor as a solution to the problem of logical determinism, generated by future contingent propositions. We will also show that the logic resulting from the abundance of the future is a non-adjunctive paraconsistent formalism based on subvaluations, which has the virtue that, as for supervaluationism, all classical laws are valid in it, while no formula like "A and not-A" is satisfiable (though both A and not- A can be true in a model). The semantic notion of consequence of abundant logic also shares most salient features with Jaśkowski's system of Discussive logic (see Jaśkowski 1969).

An usual argument for supervaluationism is that it is a logically "much reasonable" approach, since it is more conservative than subvaluations w.r.t classical logic and preserves a bigger deal of its demonstrative patterns. However, A. Varzi (2007) showed that, under the most natural reading of the notion of logical consequence for supervaluationism, many important classically sound inference rules fail to hold. In our paper we instead demonstrate that some of these rules, like conditional proof , argument by cases and reductio ad absurdum, hold in our logical framework and thus the two approaches (supervaluationism and the abundance of the future) are, from a logical point of view, much more on a par. From a general point of view, following the line of (Hyde 1997), we will outline a global comparison between supervaluations and abundance and their

respective virtues w.r.t. preservation of classical inference modes.

As a philosophical point in favour of abundance, we will also show that the abundance of the future can guarantee a better treatment of linguistic activities such as forecasts and bets. Such activities can be read as stating what will happen in the most probable development of the events. We shall give a semantics to such a notion, and we will show that "truth in the most probable history" is a species of abundant future, while it is in no way a species of supervaluationist future. Then, abundance proves to be a suitable candidate to a conceptually uniform approach to contingent statements about the future.

- N. Belnap and M. Green, Indeterminism and the thin red line, in Philosophical Perspectives 8: Logic and Language (J. Tomberlin ed.), pp. 217-244, Atascadero, 1994.
- Cobreros, Pablo . Supervaluationism and logical consequence: A third way. Studia Logica 90 (3), 2008
- D. Hyde, From heaps and gaps to gaps of gluts, Mind, 106, pp. 641-660, 1997
- Jaśkowski, Stanisław. Propositional calculus for contradictory deductive systems. Studia Logica 24 (1), 1969.
- G. Restall, Lukasiewicz, Supervaluations and the Future, Logic and Philosophy of Science, 3, 1-9, 2005
- R. Thomason, Indeterminist Time and Truth-Value Gaps, Theoria, 36, pp. 264-281,1970
- R. Thomason, Combinations of Tense and Modality, D. Gabbay and F. Guenther (eds.), Handbook of Philosophical Logic. Dordrecht: Reidel, 135-65, 1984.
- A. Varzi, Supervaluationism and Paraconsistency, in Frontiers in Paraconsistent Logic (D. Batens, C. Mortensen eds.) , Baldock: Research Studies Press, pp. 279-297, 2000.
- A. Varzi, Supervaluationism and its logics, Mind, 116 (3), pp. 633-675, 2007.