Applied Mathematical Concepts

Philosophers have standardly understood the application of mathematics to consist in the bridging of an empirical structure and a mathematical structure by means of (normally, structure-preserving) mappings. The mathematical structure is the model of the empirical structure and the inferences that rely upon its properties can be used in order to gain information about the empirical structure. This inferential point of view on applications has been presented in a sophisticated form by Bueno and Colyvan in a recent paper (Bueno and Colyvan 2011) but the emphasis on the central role of mappings may be found in the work of Chris Pincock (e.g. in Pincock 2004) and has informed much philosophy of applied mathematics, notably through the work of Suppes (e.g. Suppes 2003) and the use of a representational approach to mathematical modelling made in Field 1980.
Even though the authors just mentioned provide distinct accounts of applications, all of them rely upon the presupposition that an empirical structure satisfying certain formal properties is given and that it is embeddable in some mathematical structure by means of a suitable mapping.
This view of applications is very natural but it does not appear to be immediately generalizable to a type of application that frequently emerges in social science and requires the analysis of design problems, which may not depend on the presentation of an empirical structure or on the use of mappings. These problems can typically be tackled by the introduction of concepts that provide a systematic method to study them. In such cases, the application of mathematics is the introduction of concepts and arguments that act directly on the elements of the empirical problem at hand, not an application of mathematical structures *via* mappings.
This conclusion can be clearly illustrated by examining the ultrafilter proof of Arrow's theorem and some of its generalizations (e.g. Wilson 1972, Gibbard 1973 and Satterthwaite 1975), regarded as (negative) solutions to empirical design problems: the general issue is to decide whether, under prescribed conditions, there exists rules to aggregate preferences that differ from dictatorships.
The way in which mathematics is applied to deal with this problem hinges essentially on the introduction of an empirical concept, that of a decisive coalition, that turns the family of agents expressing the preferences to be aggregated into an empirical space whose properties describe the distribution of deliberative power among the agents. These properties, can be used to frame an argument to the effect that the deliberative power is concentrated in the hands of a single agent, i.e. the dictator.
This kind of result is not obtained by semantically bridging an empirical and a mathematical structure but rather by spelling out the consequences of an empirical concept, which guides the analysis of the aggregation problem. Moreover, what is being analysed is not so much an empirical structure as a rule that can be applied to a number of distinct empirical setups, some of whose properties are fixed. It is unnecessary, in this context, to seek to embed the empirical setups into mathematical models, semantically conceived: the crucial step is rather the interaction between the information available in the position of the problem and an additional concept that allows the direct deductive articulation of that information.
This suggests that the philosophical study of the application of mathematics should pay a greater attention to the varieties of contexts and the diversity of ways in which mathematics is used to deal with empirical problems.

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