Three Types of Uncertainty

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1   Introduction

Using Savage's Subjective Expected Utility (SEU) framework, this paper advocates the distinction between three types of uncertainty: Credence uncertainty, option uncertainty, and state space uncertainty.

The first, *credence uncertainty*, is that uncertainty identified in the Ellsberg paradox (Ellsberg, 1961), which has been discussed in the literature as "ambiguity" (Dekel 1986; Gilboa and Schmeidler 1989, 1992; Nehring 2001; Maccheroni, Marinacci and Rustichini 2006). It refers to the case where the agent cannot assign a unique prior probability to each state of the world.

The second, which I call *option uncertainty*, refers to the case where the state space is insufficiently fine-grained. In this case, the consequence of each act at each state is not unique. An axiomatic model for this type of uncertainty has recently been provided by Walker and Dietz (2011).

Finally, *state space uncertainty* – the most severe type of uncertainty – is the case where the state space is not exhaustive. Under that type of uncertainty, unforeseen contingencies may occur (see Kreps, 1992). In the following

sections, I will outline the main characteristics of the types of uncertainty, as well as the results of the paper.

2 Credence Uncertainty

Consider Ellsberg's (1961) experiment. An urn contains 90 balls, 30 of which are red. The remaining 60 are some mixture of black and yellow, but the exact mixture is unknown. First, the agent is asked to choose between gambles **A** and **B**:

	Red	Black	Yellow
A:	100	0	0
B:	0	100	0

Then, the agent is asked to choose between the gambles **C** and **D**:

	Red	Black	Yellow
C:	100	0	100
D:	0	100	100

It is well-known that the majority of agents express a preference of **A** over **B** as well as **D** over **C**. This is inconsistent with SEU theory since it is not compatible with a unique prior probability on states. We make several observations about this type of uncertainty:

- This type of uncertainty can be seen as *a result of the choice of partition of the state space.* Specifically, some of the consequences of the acts depend on states for which the agent has no probabilistic information; the state space is more fine-grained than the probabilistic information.

All gambles above would be unambiguous if the state space had been partitioned into the coarser states ("Red", "Black ∨ Yellow"). This partition has unambiguous probabilities, where "Red" comes about with a probability of 1/3, and "Black or Yellow" comes about with a probability 2/3.

- The agent has a *small world decision environment* at hand: He knows the relevant state space, the available actions and the outcomes of each act associated with each state. He merely lacks the relevant probabilistic information to make optimal decisions.

- The situation is one of *subjective uncertainty*: The number of red, black and yellow balls is pre-determined before the experiment takes place. If the agent were faced with the gambles several times, he could work out the probabilities of each colour. Hence, the uncertainty is purely *epistemic*.

3 Option Uncertainty

Suppose you are faced with the choice between staying in your current job and taking up a new job. Then it may be very hard for you to assess the consequences of taking up the new job: Will the projects be interesting? Will your colleagues be friendly? Such a situation is one of *option uncertainty*: The agent is not sure of the consequences of each act at each state. I argue that this type of uncertainty is both theoretically and intuitively distinct from credence uncertainty: It cannot be reduced to credence uncertainty.

Specifically:

Proposition 1: *Option uncertainty is that uncertainty over consequences which is causally brought about by the choice of act the agent makes. Option uncertainty which is not causally brought about by the agent's choice of act can be reduced to credence uncertainty.*

The proposition follows from the assumption of state-act independence in Savage's framework: The agent's choice of act should not affect the likelihood of the states. I show that option uncertainty cannot be reduced to credence uncertainty, since otherwise state-act independence will be violated.

4   State Space Uncertainty

In Savage's framework, the state space is a primitive of the theory. It is designed to capture all decision-relevant contingencies on which the agent's choice of act may depend. However, such an exhaustive state space may not be readily available to the agent, or hard to construct. In this case, the agent may have omitted a state of the world. Unforeseen contingencies may then occur.

Proposition 3: *State space uncertainty is that uncertainty over states of the world which is brought about by unforeseen contingencies. State space uncertainty arising through an excessively coarse specification of the states of the world can be reduced to credence uncertainty.*

I argue that in the case of state space uncertainty, optimal solutions are no longer feasible. This is so since the expected utility ordering between acts could be reversed, had the unforeseen contingency been taken into account.

It is not clear how a rational agent should deal with a case of state-space uncertainty; perhaps this is the case where Savage's mantra "look before you leap" ceases to be applicable, and the agent must content himself to "cross the bridge when he comes to it".

5   Conclusion

This paper advocates a new taxonomy of uncertainty. As I show, the types of uncertainty are distinct on a theoretical level and cannot be treated similarly. The distinction I proffer has both normative and descriptive implications. First, one may ask the question whether a rational agent should behave differently, depending on the type of uncertainty he is faced with. Second, one may investigate empirically whether real world agents treat the types of uncertainty identified here differently. I conjecture that this is indeed the case; this question is open for further investigation.