REFERENCE POINTERS AND A HYBRID LANGUAGE FOR ACTUALISM

Text

Actualism is the view that actual objects and "mere possibilia" are not ontologically on par: the identity (or existence) conditions for actual objects would be clear and unproblematic, while the ones of mere possibilia would be puzzling and far from perspicuous. A formal way to render the actualist bias is the standard actualist language **A**. Such a language expands the standard modal propositional language **ML** with the actualist universal quantifier Π and its dual Σ (the existential actualist quantifier). The two quantifiers range at any world w just on the objects that exist in w. The opponents of actualism put actual objects and mere possibilia on an ontological par and use the standard possibilist language **P**, by expanding **ML** with the possibilist universal quantifier $\forall$ and its dual $\exists$ (the existential possibilist quantifier). $\forall$ and $\exists$ range at any world w on all the possible objects (i.e. on those objects that exist in some possible world w'). It is well known that **A** is less expressive than **P**: all sentences in **A** have a translation in **P** and a back-translation from **P**, while the converse does not hold.

Correia proves in [2] that there are actualist and possibilist languages ($\mathbf{A_V}$ and $\mathbf{P_V}$, respectively) that are equally expressive. $\mathbf{A_V}$ ($\mathbf{P_V}$) is **A** (**P**) supplemented with pairs of reference-pointers $\{\uparrow_i, \downarrow_i\}$. The function of $\uparrow_i$ ($\downarrow_i$) is to fix (retrieve) reference to a given world, according to a store list s that orders worlds in a sequence. In a model M_V with interpretation σ, the truth-clauses for the non-quantified propositions $\uparrow_i p$ and $\downarrow_i p$ are as follows: $M_V^\sigma, s, w_i \models \uparrow_i p$ iff $M_V^\sigma, s(i \rightarrow w_i) \models p$ and $M_V^\sigma, s, w_i \models \downarrow_i p$ iff $M_V^\sigma, s, s(i) \models p$, where $s(i \rightarrow w_i)$ is the result of replacing the i-th world in s by w_i and $s(i)$ is the i-th world in s. The relevant cases for the expressivity result of the two languages are those involving Π (Σ) and $\forall$ ($\exists$). [2] shows that there is a translation τ from $\mathbf{A_V}$ to $\mathbf{P_V}$ and such that $\tau(\Pi x\, p) = \forall x(Ex \rightarrow \tau(p))$ and a back-translation τ' from $\mathbf{P_V}$ to $\mathbf{A_V}$ and such that $\tau'(\forall x(p)) = \uparrow_i L \Pi x \downarrow_I p$, where L is the necessity operator and p contains no quantifiers (where the latter holds, we have $\tau/\tau'(p)) = p$).

$\mathbf{A_H}$ ($\mathbf{P_H}$) is **A** (**P**) plus an infinite countable set of world-propositions Φ_i, Φ_j…. World-propositions are such that, for any proposition p, either $\Phi_i \Rightarrow p$, or $\Phi_i \Rightarrow \neg p$, where $\Rightarrow$ is strict implication. Their function is to individuate a world by implying all and only the sentences that are true in that world. In a model M_H with interpretation σ, the truth-clause for Φ_i is: $M_H^\sigma, l, w_i \models \Phi_i$ iff w_i is the i-th world in l, where the latter is a list based on W. The truth-clause for $\Phi_i \wedge p$ (where p contains no quantifiers) is: $M_H^\sigma, w_i \models \Phi_i \wedge p$ iff $M_H^\sigma, w_i \models \Phi_i$ and $M_H^\sigma, w_i \models \Phi_i \Rightarrow p$. $\mathbf{A_H}$ and $\mathbf{P_H}$ are hybrid languages analogous to the ones presented in [1] and [3]. The equivalence of the expressive powers of $\mathbf{A_H}$ and $\mathbf{A_V}$ is proved by building a validity-preserving translation τ^* from $\mathbf{A_V}$ to $\mathbf{A_H}$ and a validity preserving back-translation $\tau^{*\prime}$ from the latter to the former. The equivalence of the expressive powers of $\mathbf{A_H}$ and $\mathbf{P_H}$ is proved in a similar way.

References:

[1] Blackburn Patrick (2006), Arthur Prior and Hybrid Logic, Synthese, 150/3: 329-372.

[2] Correia Fabrice (2007), Modality, Quantification and Many Vlach Operators, Journal of Philosophical Logic, 36/4: 473-488.

[3] Prior Arthur & Fine Kit (1977) Worlds, Times and Selves, Duckworth, London.