From $(AB)a$ infer A^*a

ABSTRACT

This talk argues that a fifth rule ought to be added to the existing four rules of inference jointly defining the logic of property modification. The new rule is needed to validate one of the standard rules, but is also required when inferring, for instance, that if you have a forged banknote and a forged passport then you have two forged things. I defend the rule against three objections and demonstrate, *inter alia*, how the modifier *Very* may be construed either as a property modifier or a (higher-order) modifier modifier.

Let a property be a function from possible worlds to functions from times to sets of individuals. Let a property modifier be a function from properties to properties. A predicate such as 'is a former president' denotes the property resulting from applying the modifier *former* to the property *being a president*. Where A is a modifier, B, A^* properties and a an individual, the four standard rules are (in rudimentary notation):

Intersective. From $(AB)a$ infer $A^*a \wedge Ba$

Subsective. From $(AB)a$ infer Ba

Modal. From $(AB)a$ infer $Ba \vee \neg Ba$

Privative. From $(AB)a$ infer $\neg Ba$

The fifth rule imitates, as it were, the effect of detaching A from $(AB)a$. Actual detachment is impossible, since A is a modifier and so applicable to B but inapplicable to a. Detachment would generate a nonsensical sentence like, "a is former". But — or so I shall argue — A can be what I call *pseudo-detached* to generate a meaningful sentence like, "a is a former something". The fifth rule is:

Pseudo-detached. From $(AB)a$ infer A^*a

Pseudo-detachment, when spelt out, is this two-step *rule*:

$$\frac{a \text{ is an } AB}{a \text{ is an } (A \text{ something})}$$

$$\frac{A^* \text{ is the property } (A \text{ something})}{a \text{ is an } A^*}$$

Let $[AB]$ be the property resulting from applying A to B, and let $[AB]_{wt}$ be the result of applying the property $[AB]$ to the world and time variables w, t to obtain a set, in the form of a characteristic function, applicable to a. Further, let $=$ be the identity relation between properties, and let p range over properties, x over individuals. Then the *proof* of the rule is this:

1.	$[[AB]_{wt}\ a]$	assumption
2.	$\exists p\ [[Ap]_{wt}\ a]$	1, EG
3.	$[\lambda x\ \exists p\ [[Ap]_{wt}\ x]\ a]$	2, β-expansion
4.	$[\lambda w' \lambda t'\ [\lambda x\ \exists p\ [[Ap]_{w't'}\ x]]_{wt}\ a]$	3, β-expansion
5.	$A^* = \lambda w' \lambda t'\ [\lambda x\ \exists p\ [[Ap]_{w't'}\ x]]$	definition
6.	$[A^*_{wt}\ a]$	4, 5, Leibniz's Law

In essence, pseudo-detachment is a rule for replacing the modifier A by the property A^*. The trick is to quantify B away and replace it by the $\exists$-bound p. So if Jumbo is a small elephant, then Jumbo is small*; i.e. Jumbo is a small something (rather than small *absoluter*), because there is a property p such that Jumbo is a (small p). This is trivially valid rather than contentious or outright wrong, as soon as quantification over properties is embraced.