Humberstone's ∃-validity in the light of Dialogical Logic

In [4], L. Humberstone gives an axiomatic description of the formulas which for every model are true at some point of that model. These formulas are the consequences of the empty set, with respect to a particular consequence relation introduced by G. Priest in [6] in order to study the Jain logical tradition. We shall call this relation the "satisfiability relation"[1] and denote it as $\Vdash_s$. The formulas studied in [4] are called ∃-valid, or universally satisfiable.

We propose a double contribution related with this topic. One is to provide a proof-theory sound and complete with respect to $\Vdash_s$. This will be done in the field of the Dialogical Logic introduced by [5] and further developped in, among other works, [7], [8]. Aside from technical considerations, we will insist on how the interaction between two players in dialogical games provides an insightful grip on $\Vdash_s$.

As is noted in [4], $\Vdash_s$ is very different from standard modal consequence relations. We may however link it to recent discussions going on the notions of modal validity, necessity, and logical truth[2]. Let us be more specific. The thesis according to which all logical truths are necessary has come under some challenges. One of these consists in considering a modal language including the actuality operator (or some equivalent device) and in defending that there are formulas in this kind of language which should count as logically true despite not being necessary[3]. Positions alike are held for example in [10], or more recently in [9], and we could summarize them as defining validity as "truth for every model in the actual world of that model", and we shall denote the corresponding consequence relation as $\Vdash_@$. ∃-validity is obviously very close to this definition, except that a formula, for being universally satisfiable, has to be true at some point of every model. This crucial difference has the effect that $\Gamma \Vdash_s A$ holds even if Γ and A are not satisfied at the same point of the model. The second aim of our proposal is thus to introduce philosophical discussions emerging from the confrontation between $\Vdash_s$ and $\Vdash_@$.

[1] following Tulenheimo's terminology

[2] see for example [3]

[3] A typical example, where @ is the actuality operator, is the formula $@p \leftrightarrow p$

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