Approximating Identity Criteria

Identity criteria are used to confer ontological respectability: Only entities with clearly determined identity criteria are ontologically acceptable. From a logical point of view, identity criteria should mirror the identity relation in being reflexive, symmetrical, and transitive. However, this logical constraint is only rarely met. More precisely, in some cases, the relation representing the identity condition fails to be transitive. Consider e.g. an intuitively plausible identity criterion for colours: a colour sample x has the same colour than a colour sample y iff they are perceptually indistinguishable in colour. It is easy to verify that the relation of perceptual indistinguishability is not transitive: given three colour samples x, y, and z it can be the case that we perceive x and y as indistinguishable in colour, and the same for y and z, but we perceive the colour of x slightly different from the colour of z. So, the relation of perceptual indistinguishability should be judged as logically inadequate for being an identity condition for colours. Williamson [1], [2] and De Clercq and Horsten [3] propose not to give up the non-transitive identity conditions that we intuitively use. They rather suggest defining equivalence relations that approximate them. Consider the following formulation of identity criteria:

$$\forall x \forall y\ ((K(x) \wedge K(y)) \rightarrow (x = y \leftrightarrow R\ (x, y)))$$

Given a non transitive identity condition R for individuals belonging to some sort K, Williamson suggests considering an equivalence relation that is either a sub- or a super-relation of R. De Clercq and Horsten, instead, propose to search for an equivalence relation $R^{\pm}$ that partially overlaps R. The latter approach has the advantage to generate closer approximations to R than Williamson's approaches.

We focus on De Clercq and Horsten's proposal and expand their formal framework by taking into account two further aspects that we consider essential in the application of identity criteria to obtain logical adequacy: *contexts* and *granular levels*. Contexts are taken to be groups of elements of the domain, and granular levels standards of precision by which observing objects in a certain context. Informally, our suggestion is as follows: Given a fixed context, each granular level provides a relation R for that context; however, if we fix a granular level of observation, R can hold between two objects in a context and not hold between the same objects in a different context. We sketch a formalisation of such suggestion. Let D be a fixed, non-empty domain of objects. A context o is defined as a subset of the domain D. So, the set of all contexts O in D is the powerset of D: $O = \wp(D)$. Consider now a binary relation R (a two-arity predicate). Assume that R is reflexive and symmetric, but not necessarily transitive. R pairs the elements in each context $o \in O$ that are indistinguishable in some respect. For instance, in the case of colour samples, R gives rise to a set of ordered pairs, each of them consisting of elements that are indistinguishable with regard to their (perceived) colour. We want R to vary across contexts as well as across granular levels. Consider, firstly, granular levels. R behaves in a specific way in each context $o \in O$ in each granular level. Take the following context with three elements: $o = (a, b, c)$. One of the following scenarios can occur: (1) R gives rise to three ordered pairs. (2) R gives rise to two ordered pairs. (3) R gives rise to one ordered pair. (4) R does not give rise to any ordered pair. We can understand the different behaviour of R in the scenarios (1)–(4) if we think of each scenario as a description of the context o given in a specific level of observation. For example, in (1), we are in a coarse-grained level; in (4), in a very fine-grained level; and in (2) and (3), in some intermediate granular level. The same can be done for all contexts $o \in O$. Now, call *context structure* a structure M consisting of the domain D, all the contexts in D, and a binary relation R (a two-arity predicate); formally, $M = \langle D, O, R \rangle$. We have seen that, in a fixed domain and set of contexts, R can vary across granular levels. More precisely, we have more than one context structure: There is at least one context structure for each granular level. Consider again the scenarios (1)–(4). We have some very coarse context structures with an R that behaves as in (1), some refined context structures with an R that behaves as in (4), and other context structures with an R that behaves as in (2) or (3). Now, consider the behaviour of R across contexts. Fix a context structure, say M_1. Consider two contexts: $o = (a, b, c)$, $o' = (a, b, c, d)$. Suppose that M_1 has a relation R such that $R(a,b)$ and $R(b,c)$ in o, and $R(a,b)$ in o'. You can observe that R holds between b and c in o, but it does not hold between them in o'. So, fixed a context structure, a relation R can vary across contexts. If, according to some context structure, the relation R fails to be transitive with respect to some context $o \in O$, then we can define an equivalence relation $R^{\pm}$ as suggested by De Clercq and Horsten. In contexts where R is not transitive, $R^{\pm}$ denotes a relation that differs from R in that it adds and/or removes some ordered pairs to or from R.

References

[1] T. Williamson, Criteria of Identity and the Axiom of Choice. *The Journal of Philosophy* 83(7), 1986, 380-394.
[2] T. Williamson, *Identity and Discrimination*. Blackwell 1990.
[3] R. De Clercq and L. Horsten, Closer. *Synthese* 146(3), 2005, 371-393.