Philosophy in practice, problem complexity and hard instances, or Is complexity a collective effect?

Field : general philosophy of science

As emphasized by philosophers like Paul Humphreys or Bill Wimsatt, philosophers should pay attention not to what is possible in principle but to what is possible in practice, at least if they are to describe which results or phenomena we do, can or may have the knowledge of. Indeed, knowing that something is predictable (resp. provable, explainable, computable, etc.) in principle while it is perhaps not predictable (resp. provable, etc.) in practice is of little to determine whether it can be actually predicted (resp. proved etc.).

Since there already exists a theory that describes how much resources are needed to solve problems, namely computational complexity theory, it is natural to resort to this theory to determine whether something can be predicted, proved, etc. in practice. And since this theory says that some problems (e.g. NP-complete problems) are intrinsically difficult to solve (for example, finding the ground state of a Ising spin glass is NP-complete), it is legitimate to expect that (P): the corresponding predictions (resp. proofs, etc.) are intrinsically difficult. My talk is devoted to showing that things are not so simple.

I first argue that (P) is a paralogism: instances of intrinsically difficult problems can be surprisingly easy to solve. Complexity results such as NP-completeness are holistic features, which are about problems. A problem (e.g. adding numbers) is an infinite collection of instances (e.g. 2+2, 5+6). Complexity results prove that no algorithm can solve all instances of a problem quickly. But this feature of the problem does not always mirror the hardness of all its instances. Indeed, recent studies about intrinsically hard NP-complete problems have showed that the instances of these problems can be almost always easy to solve. As a result, the suspicion may rise that complexity is a collective effect and that instances, taken individually, can always be solved quickly.

In a second step, I present the notion of complexity core of a problem, which was devised in order to single out the hard instances of a hard problem. A complexity core of a problem is an infinite set of instances that is homogeneous in the sense that it does not have any infinite easy subset[1]. In spite of this welcome feature, I argue that the notion of complexity core remains a collective notion and that being an instance of a complexity core is not a sufficient condition for being intrinsically difficult.

I conclude that, in the framework of computational complexity, it does not seem to be possible to show that such or such instances of problems are intrinsically difficult. This is too bad for philosophers because, when they claim that such or such phenomena or results are unpredictable, unprovable in practice or computationally emergent, they intent to be speaking about particular phenomena or results, not about infinite sets of them. My talk concludes that, till philosophers do not provide a well-grounded notion of hard instance, such discussions or concepts are built on shaky foundations.

[1] Note the similarity with the notion of objectively homogeneous class in Salmon's SR model.